R. van der Veen Fruehlingssemester 2008

Exercises Knot Theory
4. Aufgabenblatt

Exercise 1. Show directly that $T(p, q)$ is equivalent to $T(q, p)$.

Exercise 2. Let $K \times L$ be the connected sum of the knots K and L. Show that $\pi_1(\mathbb{S}^3 - K \times L) = \pi_1(\mathbb{S}^3 - K) \times \pi_1(\mathbb{S}^3 - L)$.

Exercise 3. Let $G = \langle a, b | a^p = b^q \rangle$. Show that $C = \langle a^p \rangle$ is the center of G and that G/C is the free product of a cyclic group of order p and one of order q.

Exercise 4. Show that torus knots are prime.

Exercise 5. Definition. A subset K of a space X is called an n-knot in X if it is homeomorphic to $\mathbb{S}^n$ for some n. Two n-knots K and L are equivalent if there is a homeomorphism $h : X \to X$ sending K to L. Show that any two 1-knots in $\mathbb{R}^4$ are equivalent.

Exercise 6. Let K be a 1-knot in $\mathbb{S}^3$ and let $k : \mathbb{S}^1 \to \mathbb{S}^3$ be the associated embedding of the circle. Show that the suspension of k gives rise to a 2-knot in $\mathbb{S}^4$. Also show that the complement of the image in $\mathbb{S}^4$ is homotopy equivalent to the complement of K.

Exercise 7. Determine the number of components of a rational knot.

Exercise 8. Let K be the rational knot corresponding to a continued fraction with coefficients $a_1, a_2, ..., a_{2n+1}$ and let L be the rational knot corresponding to the reversed sequence. Are K and L equivalent or not?

Exercise 9. Look at the video NotKnot on youtube and browse through the wiki sytle Knot Atlas.